NOBEL PRIZE 2015 NEUTRINO OSCILLATION I

KOREA INSTITUTE FOR ADVANCED STUDY

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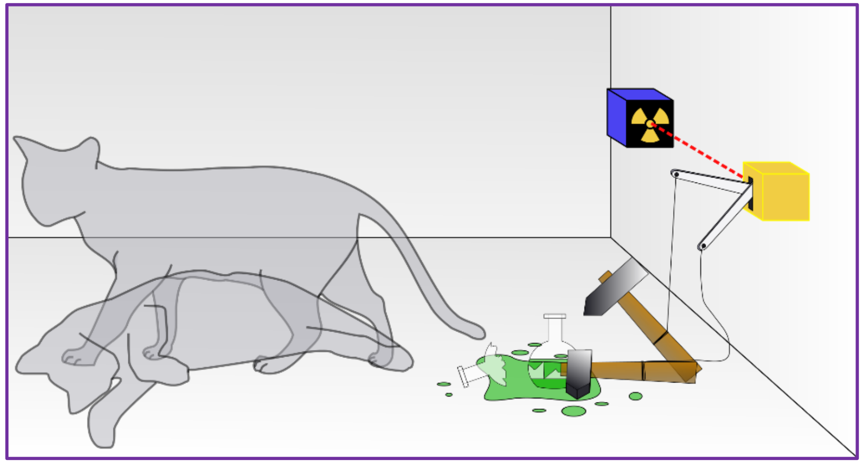
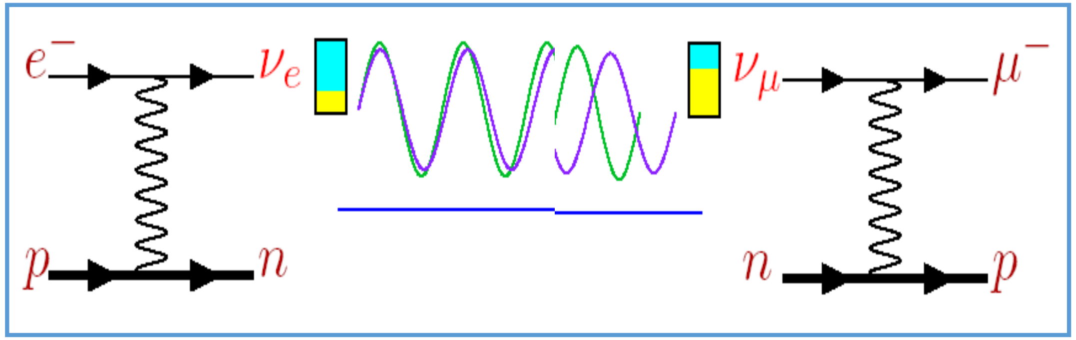
Outline

- Introduction: a two-state QM



- Weak interaction & neutrino oscillation

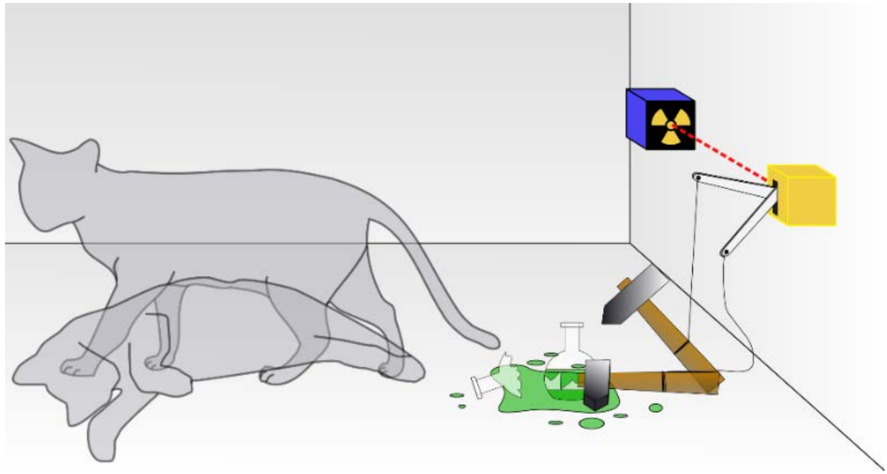


- Origin of neutrino mass

- Conclusion

A two-state problem

Quantum mechanics of a two-state system



- Schroedinger's cat (an illustration):
  Observation – **A**live(undecayed) or **D**ead(decayed)

$$\mathcal{O} : |A\rangle \text{ or } |D\rangle$$

- Hamiltonian: energy operator of the system

$$i\frac{d}{dt}|\psi(t)\rangle = \mathcal{H}|\psi(t)\rangle$$

cf) $H = E_0 - \frac{i}{2}\Gamma$

- Non-simultaneous observables: $[\mathcal{H}, \mathcal{O}] \neq 0$

- If

$$\mathcal{H}|A\rangle = E_{11}|A\rangle + E_{12}|D\rangle$$
$$\mathcal{H}|D\rangle = E_{21}|A\rangle + E_{22}|D\rangle$$

Alive or Dead at t? $|\psi_A(0)\rangle = |A\rangle \;\; \Rightarrow \;\; |\psi_A(t)\rangle = ?$

Superposition of A & D: to be or not to be?

- Energy eigenstates: $\mathcal{H}|E_{1,2}\rangle = E_{1,2}|E_{1,2}\rangle$

$$|E_1\rangle = c_\theta|A\rangle - s_\theta|D\rangle$$
$$|E_2\rangle = s_\theta|A\rangle + c_\theta|D\rangle$$

$$i\frac{d}{dt}|\psi_{1,2}(t)\rangle = E_{1,2}|\psi_{1,2}(t)\rangle \quad \Rightarrow \quad |\psi_{1,2}(t)\rangle = e^{iE_{1,2}t}|E_{1,2}\rangle$$

- The poor cat state after some time:

$$\begin{aligned}|\psi_A(t)\rangle &= e^{iE_1t}c_\theta|E_1\rangle + e^{iE_2t}s_\theta|E_2\rangle \\ &= (e^{iE_2t} - e^{iE_1t})s_\theta c_\theta|D\rangle + \cdots|A\rangle\end{aligned}$$

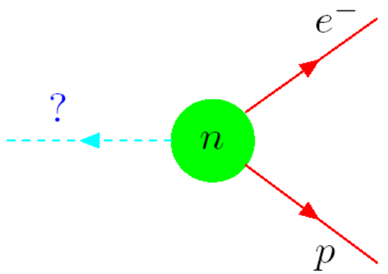
- Probability to be found dead at t:

$$P_{AD}(t) = |\langle D|\psi_A(t)\rangle|^2 = \sin^2(2\theta)\sin^2(\frac{\Delta E t}{2})$$

Weak interaction & neutrino oscillation

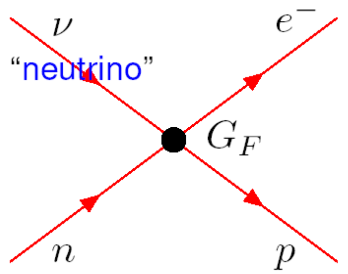
From beta decay to two neutrinos

Wolfgang Pauli (1930) :
"The unseen little neutral one"
in β–decay



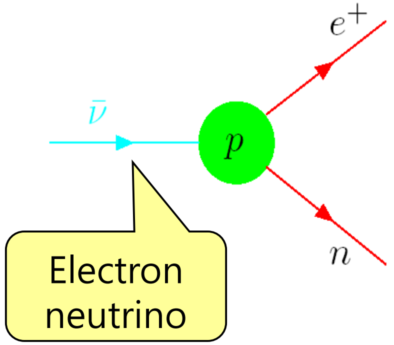
$$A^Z \to A^{Z+1} + e^- + ?$$

Enrico Fermi (1933) :
Theory of Weak Interaction



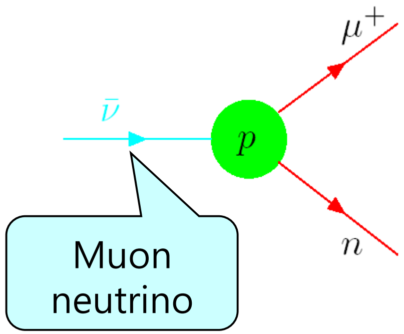
$$G_F \approx 4 \times 10^{-33} cm^2$$

Cowan & Reines (1957) :
"observed" neutrinos



⇒ Nobel prize 1995

Lederman, Schwartz & Steinberger (1961) :
"another type" of neutrinos



⇒ Nobel prize 1988

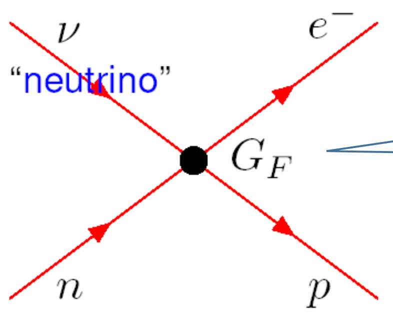
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \nu_e \neq \nu_\mu \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

Two **flavor/generation/family**

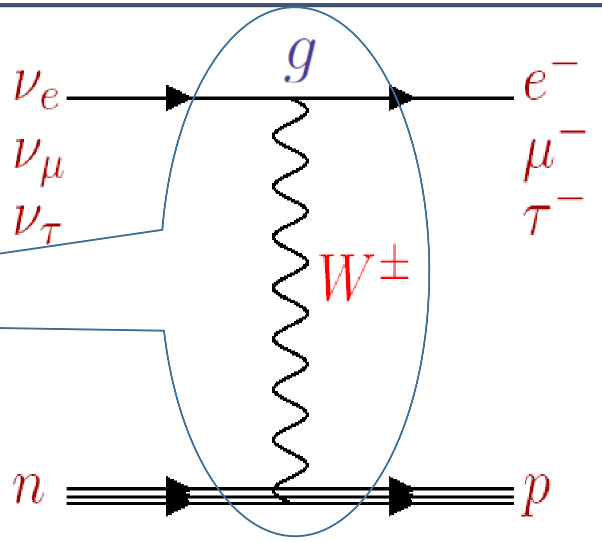
Weak interaction & Higgs mechanism

Enrico Fermi (1933) :

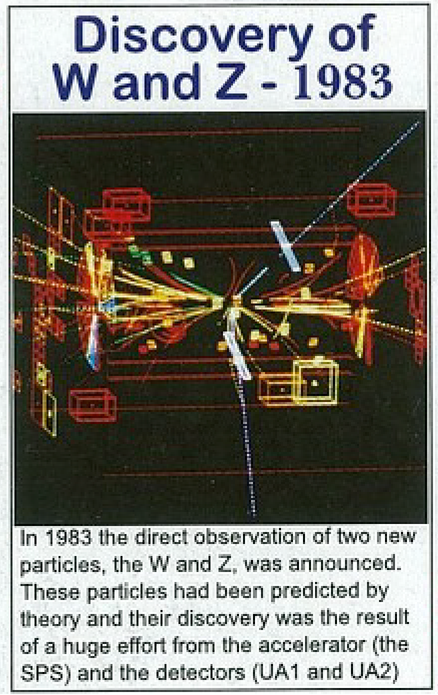
Theory of Weak Interaction



$$G_F \approx 4 \times 10^{-33} cm^2$$



"Charged Current"



$$G_F = \frac{g^2}{2\sqrt{2}m_W^2} = \frac{1}{\sqrt{2}v^2}$$

A MODEL OF LEPTONS*

Steven Weinberg†

Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 17 October 1967)

Higgs mechanism

$$\langle H^0 \rangle = \frac{v}{\sqrt{2}}$$

Standard Model

force quanta

γ

$W^{\pm}, Z^0$

G^a

Standard Model

three lepton families			Q
ν_e	ν_μ	ν_τ	0
e	μ	τ	-1

three quark families			
u	c	t	$2/3$
d	s	b	$-1/3$

H

Masses (MeV)

0	?	?	?
80,000	0.5	107	1,780
0	2	1,300	4,300
	5	170	174,000

$$m_f = y_f \langle H^0 \rangle$$

Mystery of Masses

Neutrino Oscillation

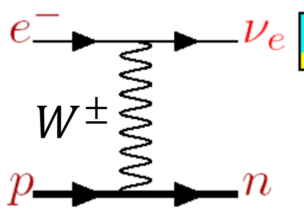
A Quantum Mechanical effect occurring when

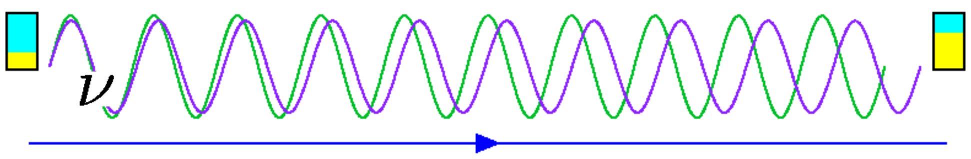
interaction eigenstates are different from mass eigenstates

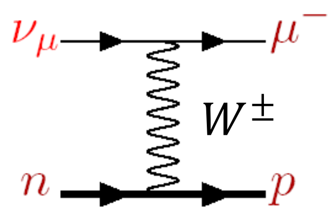
Two neutrinos

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\begin{pmatrix} m_{ee} & m_{e\mu} \\ m_{e\mu} & m_{\mu\mu} \end{pmatrix} = U_\theta \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} U_\theta^T$$







$$E_i \approx p_i \approx E, t \approx L$$

$$E_i = \sqrt{p_i^2 + m_i^2} \approx E + \frac{m_i^2}{2E}$$

Production

Detection

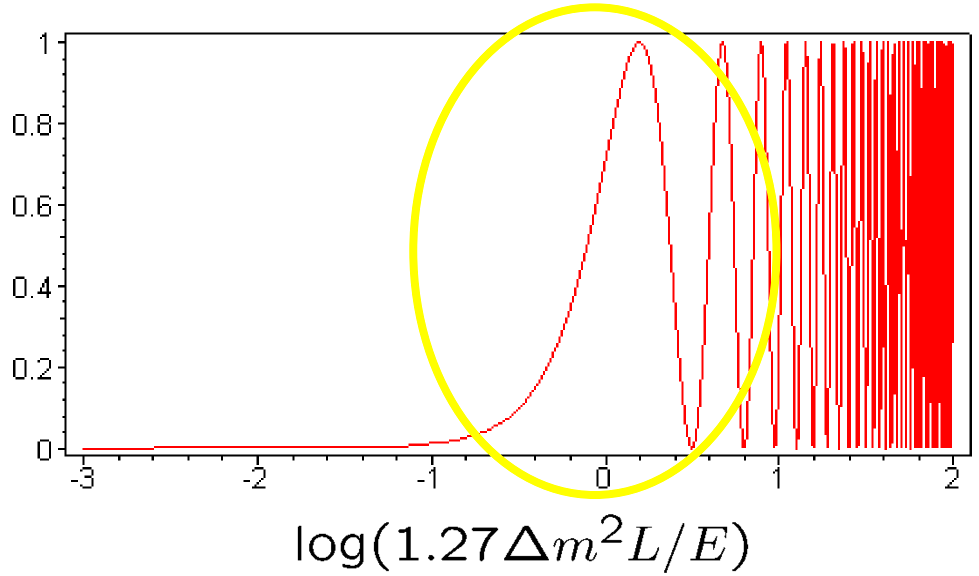
Two Neutrino Oscillation Probability

$$e^{i(E_i t - p_i L)} \approx e^{i\frac{m_i^2}{2E}L}$$

$$|\nu_e(L)\rangle = e^{i\frac{m_1^2 L}{2E}}\cos\theta|\nu_1\rangle + e^{i\frac{m_2^2 L}{2E}}\sin\theta|\nu_2\rangle$$

$$\langle\nu_\mu|\nu_e(L)\rangle = \sin\theta\cos\theta[e^{i\frac{\Delta m_{21}^2}{4E}} - e^{-i\frac{\Delta m_{21}^2 L}{4E}}]$$

$$P_{e\mu} = \sin^2 2\theta\ \sin^2\left(1.27\frac{\Delta m_{21}^2(\text{eV}^2)L(\text{km})}{E(\text{GeV})}\right)$$



Three Neutrino Oscillation

$$
\begin{aligned}
|\nu_e\rangle &= U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle \\
|\nu_\mu\rangle &= U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle \\
|\nu_\tau\rangle &= U_{\tau 1}|\nu_1\rangle + U_{\tau 2}|\nu_2\rangle + U_{\tau 3}|\nu_3\rangle
\end{aligned}
$$

$$
U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\theta_{23}} & s_{\theta_{23}} \\ 0 & -s_{\theta_{23}} & c_{\theta_{23}} \end{pmatrix}
\begin{pmatrix} c_{\theta_{13}} & 0 & s_{\theta_{13}} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{\theta_{13}} e^{-i\delta} & 0 & c_{\theta_{13}} \end{pmatrix}
\begin{pmatrix} c_{\theta_{12}} & s_{\theta_{12}} & 0 \\ -s_{\theta_{12}} & c_{\theta_{12}} & 0 \\ 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}
$$

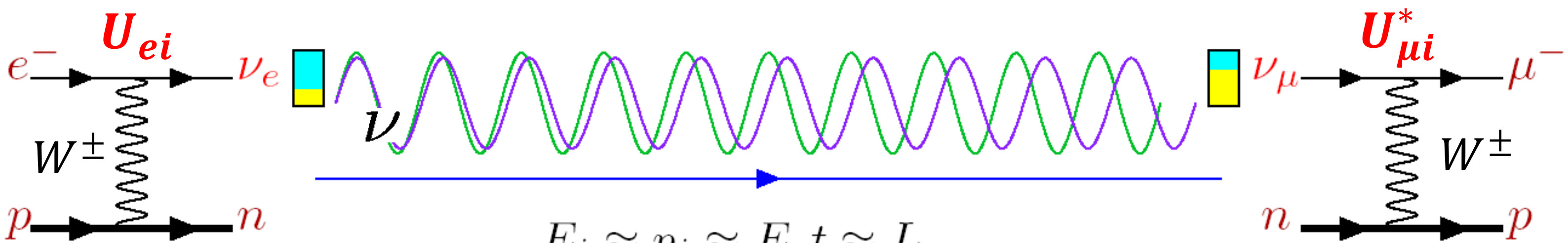
atmospheric　　reactor　　solar　　Majorana

- Three mass eigenvalues: $m_1, \; m_2, \; m_3 \Rightarrow$ Two mass^2 differences

$$\Delta m^2_{31} = m_3^2 - m_1^2 \ (\text{atmospheric}), \quad \Delta m^2_{21} = m_2^2 - m_1^2 \ (\text{solar})$$

- Three mixing angles and three phases: $\theta_{12}, \; \theta_{23}, \; \theta_{13}; \quad \delta, \; \phi_2, \; \phi_3$

Oscillation probability: general formula



Production

$$E_i \approx p_i \approx E, t \approx L$$

$$E_i = \sqrt{p_i^2 + m_i^2} \approx E + \frac{m_i^2}{2E}$$

Detection

$$A_{e\mu} \equiv A(\nu_e \to \nu_\mu) = \sum_i U_{ei} e^{-i\frac{m_i^2}{2E}L} U^*_{\mu i}$$

$$P_{\alpha\beta} = \left|A_{\alpha\beta}\right|^2 = \delta_{\alpha\beta} - 4\sum_{i>j} \Re\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\Delta m^2_{ij} \frac{L}{4E}\right)$$
$$-2\sum_{i>j} \Im\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\left(2\Delta m^2_{ij} \frac{L}{4E}\right)$$

(homework)

Neutrino oscillation→ mass & mixing

- If massless ($m_i = 0$), $P_{\alpha\beta} = \delta_{\alpha\beta}$.
- If no mixing ($U = I$), $P_{\alpha\beta} = \delta_{\alpha\beta}$.

Anti-neutrino oscillation

$$P_{\bar{\alpha}\bar{\beta}} = \left|A(\bar{\nu}_\alpha \to \bar{\nu}_\beta)\right|^2 = \delta_{\alpha\beta} - 4\sum_{i>j} \Re\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\Delta m^2_{ij} \frac{L}{4E}\right)$$
$$+2\sum_{i>j} \Im\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\left(2\Delta m^2_{ij} \frac{L}{4E}\right)$$

$$P_{\alpha\beta} - P_{\bar{\alpha}\bar{\beta}} = -4\sum_{i>j} \Im\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\left(2\Delta m^2_{ij} \frac{L}{4E}\right) \neq 0$$

Complex U
→ CP violation

Three neutrino oscillation

$$P_{\alpha\beta/\bar{\alpha}\bar{\beta}} = \delta_{\alpha\beta} - 4\Re\left(U^*_{\alpha 3}U_{\beta 3}U_{\alpha 2}U^*_{\beta 2}\right)\left(\Delta m^2_{32}\frac{L}{4E}\right)$$
$$-4\Re\left(U^*_{\alpha 3}U_{\beta 3}U_{\alpha 1}U^*_{\beta 1}\right)\left(\Delta m^2_{31}\frac{L}{4E}\right)$$
$$-4\Re(U^*_{\alpha 2}U_{\beta 2}U_{\alpha 1}U^*_{\beta 1})\left(\Delta m^2_{21}\frac{L}{4E}\right)$$
$$\mp 2\,J\left(\sin\left(2\Delta m^2_{32}\frac{L}{4E}\right) + \sin\left(2\Delta m^2_{13}\frac{L}{4E}\right) + \sin\left(2\Delta m^2_{21}\frac{L}{4E}\right)\right)$$

where $J = \Im\left(U^*_{\alpha 3}\,U_{\beta 3}U_{\alpha 2}U^*_{\beta 2}\right) = -\Im\left(U^*_{\alpha 3}\,U_{\beta 3}U_{\alpha 1}U^*_{\beta 1}\right) = \Im(U^*_{\alpha 2}\,U_{\beta 2}U_{\alpha 1}U^*_{\beta 1})$

Jarlskog Invariant – measure of CPV

(homework)

Three neutrino oscillation

- For $|\Delta m^2_{32}| \approx |\Delta m^2_{31}| \gg \Delta m^2_{21}$

$$P_{e\mu} \approx 4\left|U_{e3}U_{\mu3}\right|^2 \sin^2(\Delta_A) - 4\Re\left(U_{e1}U^*_{e2}U^*_{\mu1}U_{\mu2}\right)\sin^2(\Delta_S) - 2J\sin(2\Delta_S)$$
$$P_{e\tau} \approx 4|U_{e3}U_{\tau3}|^2 \sin^2(\Delta_A) - 4\Re(U_{e1}U^*_{e2}U^*_{\tau1}U_{\tau2})\sin^2(\Delta_S) + 2J\sin(2\Delta_S)$$
$$P_{\mu\tau} \approx 4\left|U_{\mu3}U_{\tau3}\right|^2 \sin^2(\Delta_A) - 4\Re\left(U_{\mu1}U^*_{\mu2}U^*_{\tau1}U_{\tau2}\right)\sin^2(\Delta_S) - 2J\sin(2\Delta_S)$$

where $\Delta_A \equiv \Delta m^2_{31}\frac{L}{4E}$ & $\Delta_S \equiv \Delta m^2_{21}\frac{L}{4E}$. (homework)

What we know now

Parameter	best-fit	3σ
Δm^2_{21} $[10^{-5}$ eV $^2]$	7.37	6.93 – 7.97
$\lvert\Delta m^2\rvert$ $[10^{-3}$ eV $^2]$	2.50 (2.46)	2.37 – 2.63 (2.33 – 2.60)
$\sin^2\theta_{12}$	0.297	0.250 – 0.354
$\sin^2\theta_{23}$, $\Delta m^2 > 0$	0.437	0.379 – 0.616
$\sin^2\theta_{23}$, $\Delta m^2 < 0$	0.569	0.383 – 0.637
$\sin^2\theta_{13}$, $\Delta m^2 > 0$	0.0214	0.0185 – 0.0246
$\sin^2\theta_{13}$, $\Delta m^2 < 0$	0.0218	0.0186 – 0.0248
δ/π	1.35 (1.32)	(0.92 – 1.99)
		((0.83 – 1.99))

Three observed oscillations

- For reactor neutrinos, $\nu_e \rightarrow \nu_{\mu,\tau}$

$$\Delta_A = \frac{\Delta m_{31}^2\, L}{4E} \sim \frac{10^{-3}\mathrm{eV}^2\ 1\mathrm{km}}{10^{-3}\mathrm{GeV}} \sim 1 \gg \Delta_S$$

$$P_{e\ \mu+\tau} \approx 4|U_{e3}|^2\,(1-|U_{e3}|^2)\,\sin^2(\Delta_A) = \sin^2(2\theta_{13})\,\sin^2(\Delta_A)$$

- For atmospheric neutrinos, $\nu_\mu \rightarrow \nu_\tau$

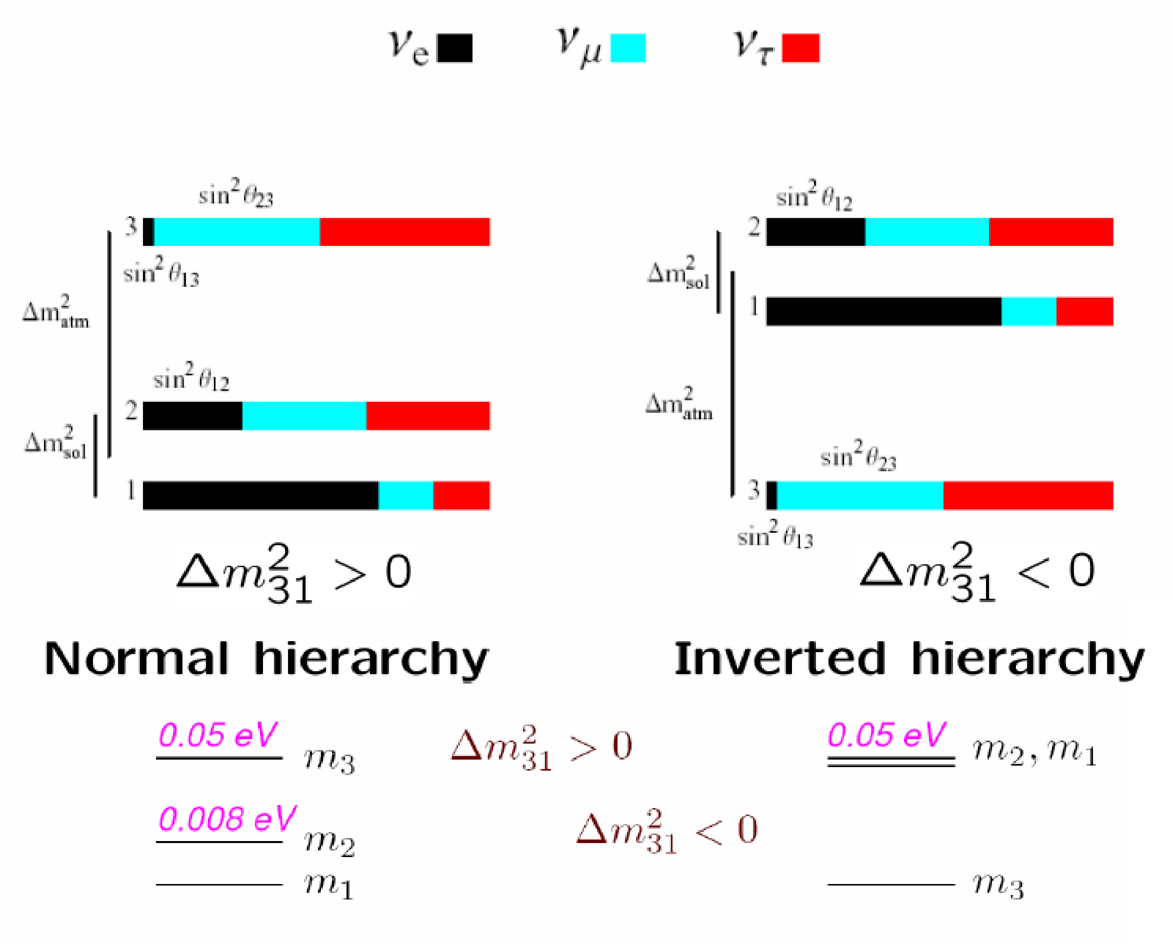
$$\Delta_A = \frac{\Delta m_{31}^2\, L}{4E} \sim \frac{10^{-3}\mathrm{eV}^2\ 10^4\mathrm{km}}{10\ \mathrm{GeV}} \sim 1 \gg \Delta_S$$

$$P_{\mu\tau} \approx 4\left|U_{\mu 3}\right|^2\ |U_{\tau 3}|^2 \sin^2(\Delta_A) = \cos^4(\theta_{13})\sin^2(2\theta_{23})\,\sin^2(\Delta_A)$$

- For Solar neutrinos, $\nu_e \rightarrow \nu_{\mu,\tau}$

$$\Delta_S = \frac{\Delta m_{21}^2\, L}{4E} \sim \frac{10^{-5}\mathrm{eV}^2\ 10^{11}\mathrm{km}}{10^{-3}\ \mathrm{GeV}} ?$$ → flavor transition in matter (MSW effect)

Unknown neutrino mass hierarchy



Quasi Degenerate

$m_1 \approx m_2 \approx m_3$
$\approx 0.05 - 0.3eV$

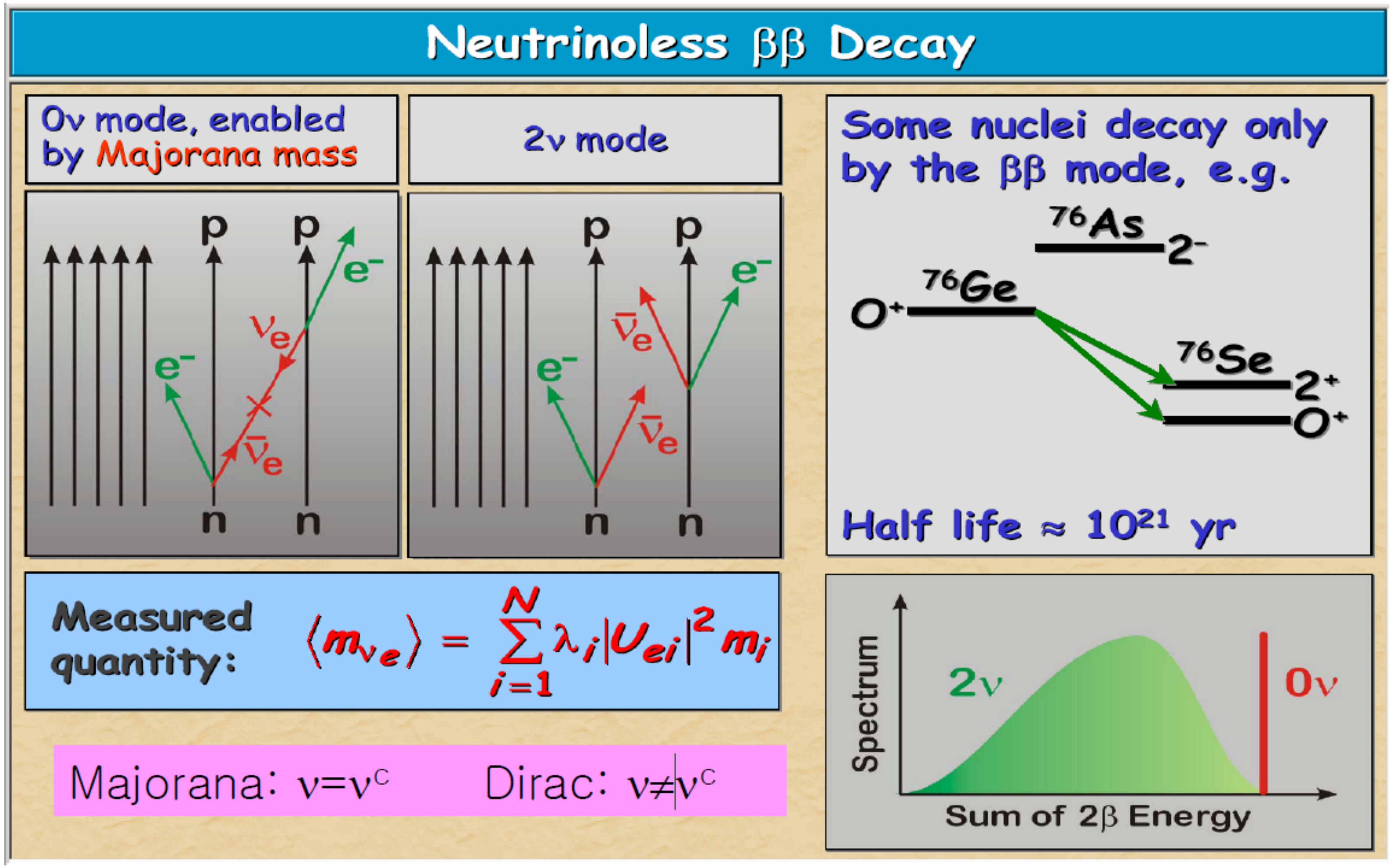
LBL experiments

- $\Delta = \Delta m^2_{31} L/4E$ $\quad L \sim 10^3 km$
- qualitative understanding $\Rightarrow$ expand in $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$ and $\sin^2 2\theta_{13}$
- matter effects $\hat{A} = A/\Delta m^2_{31} = 2VE/\Delta m^2_{31};\ \ V = \sqrt{2} G_F n_e$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - \cos^2\theta_{13} \sin^2 2\theta_{23} \sin^2\Delta + 2\,\alpha\, \cos^2\theta_{13} \cos^2\theta_{12} \sin^2 2\theta_{23} \Delta \cos\Delta$$

$$\begin{aligned}
P(\nu_e \to \nu_\mu) \approx\ & \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\
\pm\ & \sin\delta_{CP}\ \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
+\ & \cos\delta_{CP}\, \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
+\ & \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}
\end{aligned}$$

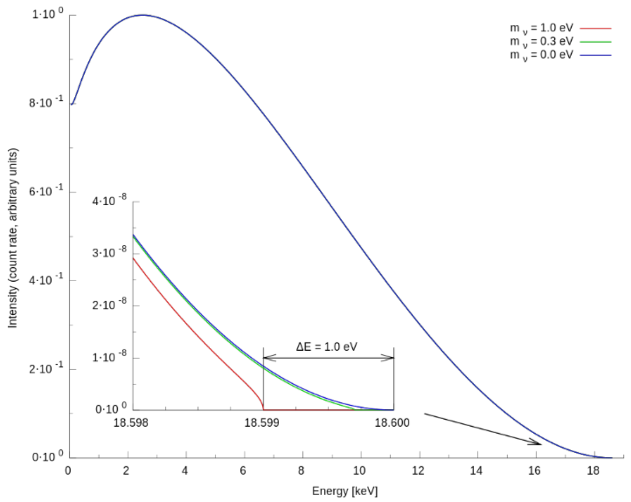
Majorana ($\nu = \nu^c$)or Dirac ($\nu \neq \nu^c$)?



Majorana mass and beta decays

$$m_\beta = \left(\sum_i |U_{ei}|^2\, m_i^2\right)^{1/2} = (c_{12}^2 c_{13}^2\, m_1^2 + s_{12}^2 c_{13}^2\, m_2^2 + s_{13}^2\, m_3^2)^{1/2},$$ β decay

$$m_{\beta\beta} = \left|\sum_i U_{ei}^2\, m_i\right| = |c_{12}^2 c_{13}^2\, m_1 + s_{12}^2 c_{13}^2\, m_2\, e^{i\phi_2} + s_{13}^2\, m_3\, e^{i\phi_3}|\ .$$ 0νββ decay



$m_\beta < 2.05 - 2.3\text{eV} \Leftarrow$ Troitsk, Mainz

$< 0.2\text{eV} \quad \Leftarrow$ KATRIN

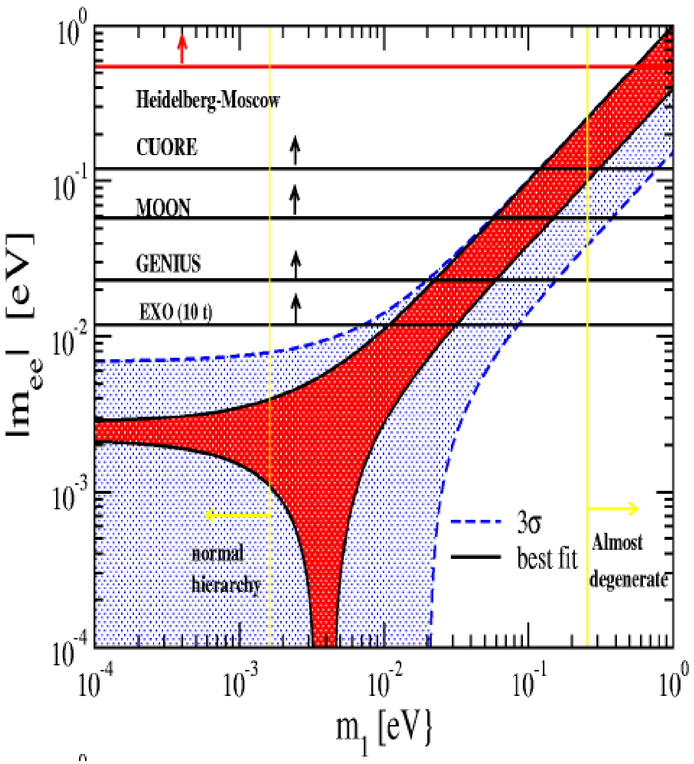
Degenerate mass: $m_\beta = m_0 < 2.0\ (0.2)\text{eV}$

$$0.056\,(0.095)\ \text{eV} \lesssim \sum_i m_i \lesssim 6\ \text{eV}$$

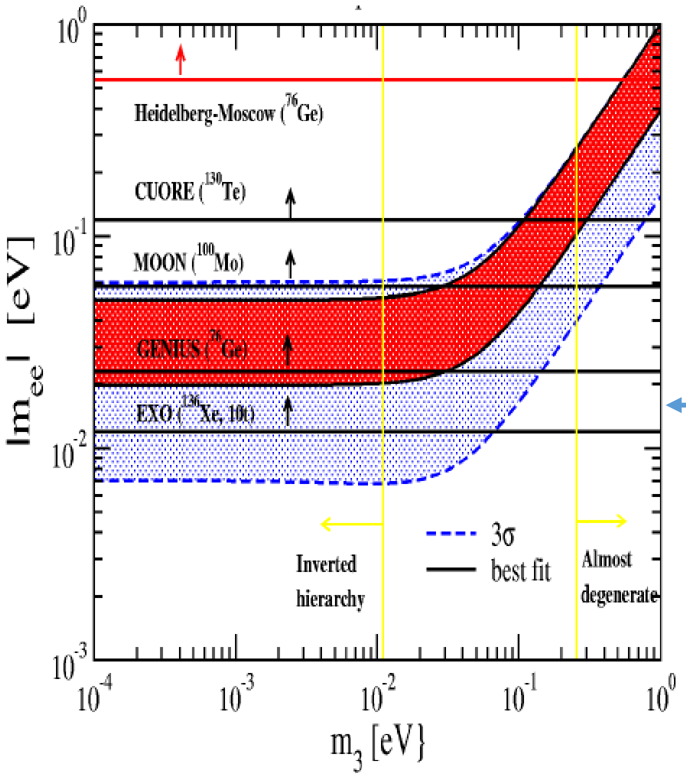
$\sqrt{\Delta m^2_{\text{atm}}}$ $3m_\beta$

$$|m_{\beta\beta}| < (0.44 - 0.62)\, h_N\ \text{eV}$$

0nbb limits



Normal hierarchy



Inverted hierarchy

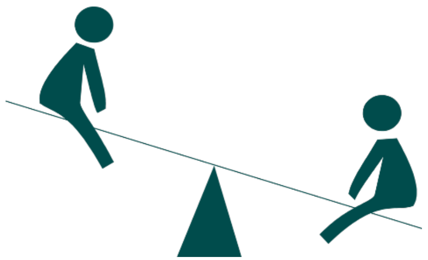
AMoRE

Origin of tiny neutrino mass?

- Introduce a SM-charge neutral fermion N to form a mass matrix with the SM neutrino ν :

$$\begin{matrix}(\nu, & N)\end{matrix} \qquad \qquad \begin{matrix}(\nu_L & \nu_H)\end{matrix}$$

$$\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{m_D^2}{M} & 0 \\ 0 & M \end{pmatrix}$$



$m_D = y_\nu\, v$

- If $M = 0,\ m_\nu = m_D \sim 0.1\ \mathrm{eV}$ $\qquad y_\nu \sim 10^{-12}$ (Dirac)
- If $m_D \sim 100\ \mathrm{GeV},\ M \sim 10^{14}\ \mathrm{GeV} \rightarrow m_\nu \sim 0.1\ \mathrm{eV}$ $\qquad y_\nu \sim 1$ (Majorana)

  If M$\sim 100\ \mathrm{GeV},\ m_D \sim 10^{-4}\ \mathrm{GeV} \rightarrow m_\nu \sim 0.1\ \mathrm{eV}$ $\qquad y_\nu \sim 10^{-6}$

Conclusion

- Oscillation – a novel quantum phenomenon.
- Nature is kind enough to exhibit observable neutrino oscillation phenomena and thus let us know the existence of neutrino masses and mixing.
- Our task is to find out the origin of tiny neutrino masses.